Exercise 23

Boyle's Law states that when a sample of gas is compressed at a constant temperature, the product of the pressure and the volume remains constant: $PV = C$.

(a) Find the rate of change of volume with respect to pressure.

(b) A sample of gas is in a container at low pressure and is steadily compressed at constant temperature for 10 minutes. Is the volume decreasing more rapidly at the beginning or the end of the 10 minutes? Explain.

(c) Prove that the isothermal compressibility (see Example 5) is given by $\beta = 1/P$.

Solution

Part (a)

Solve Boyle's law for V.

$$V = \frac{C}{P}$$

Take the derivative with respect to P.

$$\begin{aligned}\frac{dV}{dP} &= \frac{d}{dP}\left(\frac{C}{P}\right) \\ &= -\frac{C}{P^2}\end{aligned}$$

Part (b)

According to this formula, when the pressure is low, the rate that volume changes is very high; and when the pressure is high, the rate that volume changes is very low. Consequently, the volume is decreasing more rapidly at the beginning of the 10 minutes.

Part (c)

The isothermal compressibility is defined in Example 5 on page 228 as

$$\begin{aligned}\beta &= -\frac{1}{V}\frac{dV}{dP} \\ &= -\frac{1}{\frac{C}{P}}\frac{d}{dP}\left(\frac{C}{P}\right) \\ &= -\frac{P}{C}\left(-\frac{C}{P^2}\right) \\ &= \frac{1}{P}.\end{aligned}$$